LABORATORY OF ELECTROACOUSTICS

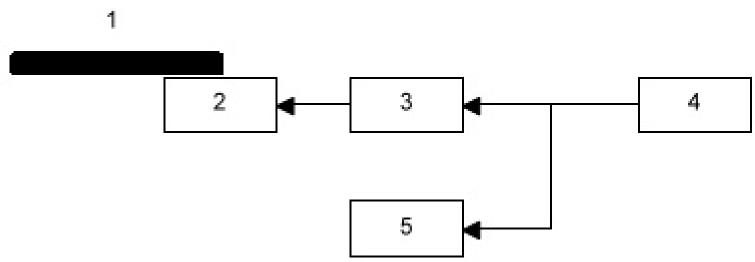
EXERCISE 1.

Vibration of mechanical systems.

The purpose of the exercise:

The aim of the exercise is learning about the properties of vibrating systems and measurement methods as well as vibration analysis. As part of the exercise, measurements of parameters describing transverse vibrations of bars clamped at one end and free ones at the other will be made.

I. Measuring system.



1 - examined vibrating system, 2 - vibration exciter, 3 - power amplifier, 4 - generator, 5 - frequency meter.

II. Laboratory tasks

2.1. Measure the frequency of transverse own vibrations (modes) of beams of different lengths and cross-sections, made of different materials.
2.2. Determine the first three ways (modes) of transverse vibrations of beams.
2.3. Obtain the obtained results in the table according to the formula (table 3) and compare with the theoretical results using the relations given in Appendix A and material data given in Tables 1 and 2.

3. Issues to prepare

3.1. Vibrations of strings, rods, beams and plates.
3.2. Natural frequencies (modes).

Literature

[1] Dobrucki A., Fundamentals of acoustics. PWR script, Wroclaw 1987.
[2] Januszajtis A., Physics for Polytechnic, Volume III Waves, §5. PWN W-wa 1991.
[3] Żyszkowski Z., Fundamentals of electroacoustics, 3rd edition. WNT W-wa 1984, ch.6.

Tabel 1. Material data.

Material	Coefficient of elasticity longitudinal (Young's modulus) E [N/m^2]	Density ρ [kg/m3]	Wave speed longitudinal (sound) c_L [m/s]
Steel	2,2 10^{11}	7800	5900
Brass	1,0 10^{11}	8600	3830
Duralumin	0,7 10^{11}	2700	5982
Plexiglas	4,45 10^{9}	1180	2670

Table 2. Moments of inertia of cross-sections with different shapes.

Cross section shape transverse	Rectangular (a x b)	Circular (r)	Equilateral triangle (a)
Moment of inertia cross-section [m^4]	$a^3b/12$	$\pi r^4/4$	$\frac{\sqrt{3}}{96}a^4$

Table 3. Measurement and calculation results for one measured object.

Material	Dimensions	n	$f_{n,meas}$ [Hz]	$f_{n,calc}$ [Hz]	δ_f [%]	m	$x_{nm,meas}$ [mm]	$x_{nm,calc}$ [mm]	δ_x [%]
	l = …….;	1				-	-	-	-
	a = …….;	2				1			
	b = …….;	3				1			
	r = …….;					2			
		4				1			
	S = …….;					2			
	I = …….;					3			

The indications in table are as follow:
l – lenght [m];
a – thickness [m];
b – width [m];
r – radius [m];
S - cross-section area [m^2];
I - moment of inertia of the cross-section [m^4];
n - vibration mode number;
m - vibration node number;
$f_{n,meas}$ – measured frequency for mode n;
$f_{n,calc}$ – calculated frequency for mode n;
$x_{nm,meas}$ – measured distance from node m to clamped site of bar for mode number n;
$x_{nm,calc}$ – calculated distance from node m to clamped site of bar for mode number n;

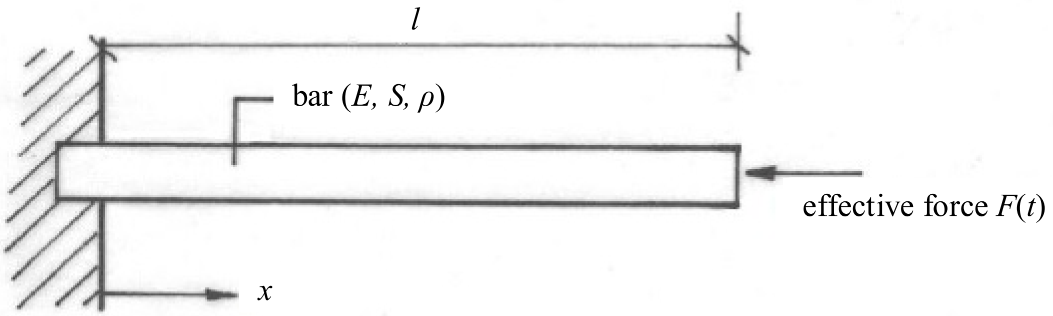
Formulas for calculating error are as follow:
$\delta_f = (f_{n,meas} - f_{n,meas}) / f_{n,meas} \times 100\%$ - frequency error for mode n;
$\delta_x = (x_{nm,calc} - x_{nm,meas}) / x_{nm,meas} \times 100\%$ - distance error for mode n and node m;

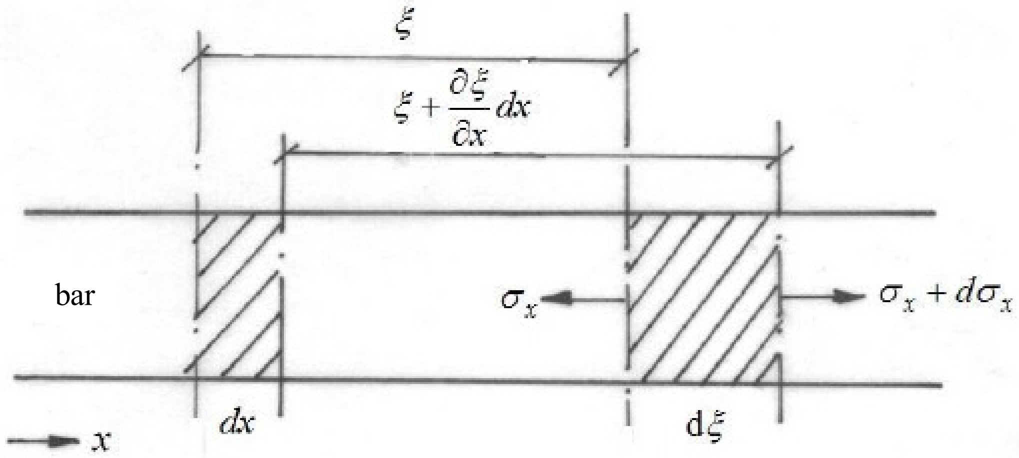
Appendix A

BAR OSCILLATION

- We consider a bar with a uniform cross-section S [m^2], made of material with a density ρ [kg/m^3] and the elasticity coefficient of the longitudinal material (Young's modulus) E [N/m^2].
- Unlike strings, the tension is not taken into account. It is assumed that the total force returning the rod to the balance position comes only from its own resilience.
- The rod can vibrate longitudinally, transversally and torsionally (vortex).

I. LONGITUDINAL BAR OSCILLATION.





Under the influence of the force F, the distance ξ between any two cross-sections has increased by $d\xi = \frac{\partial \xi}{\partial x} dx$, so the relative elongation of the bar in this area is $\varepsilon = \frac{\partial \xi}{\partial x}$, and according to Hooke's law, proportional to the tension σ_x:

$$\varepsilon = \frac{\sigma_x}{E} = \frac{1}{E}\frac{F}{S}, \Rightarrow F = \varepsilon E S.$$

The force acting on the right cross-section is equal:

$$F + \frac{\partial F}{\partial x} dx = F + ES \frac{\partial^2 x}{\partial x^2} dx,$$

that is, the final force on the bar section dξ is:

$$dF = ES\frac{\partial^2 x}{\partial x^2}dx.$$

It is the force of elasticity. Under the influence of this force, the mass of the bar section *dx* equal *dm* = *Sρdx*, is accelerated $\frac{\partial^2 \xi}{\partial t^2}$. Thus, based on Newton's second law, we get the equation:

$$\frac{\partial^2 \xi}{\partial t^2} = c_L^2 \frac{\partial^2 \xi}{\partial x^2}, (1)$$

where $c_L = \sqrt{E/\rho}$ [m/s] is the speed of the longitudinal wave (sound) in the rod.
It is a wave equation, one-dimensional (sound wave in the rod). Any type of function *ξ(x+ct)* meet the condition.

To determine the own vibrations of the rod, the procedure is similar to that of the string, i.e. the method of separating the variables. The solution is to solve the equation (1) in the form of the product of two functions, one of which depends only on x and the other only on t.

$$\xi(x,t) = X(x)T(t)$$

Finally, the solution of the wave equation is:

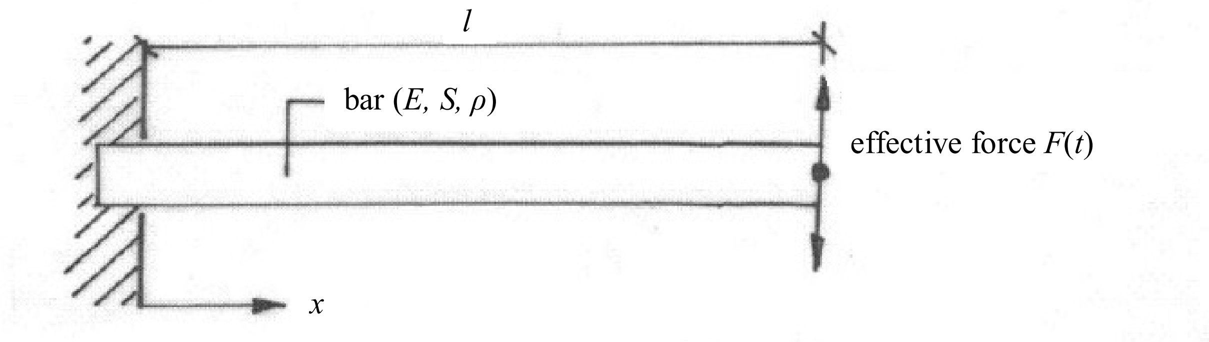
$$\xi(x,t) = \sum_n (C_{1n} cos\omega_n t + C_{2n} cos\omega_n t) sin\frac{n\pi}{l}x, (2) \;\; n = 1,2, ...$$

where the constants C_{1n}, C_{2n} are determined from the initial conditions, and the natural frequencies (modes) are equal to:

$$\omega_n = \frac{n\pi}{l}\sqrt{\frac{E}{\rho}}, (3) \; \left[\frac{rad}{s}\right].$$

It should be noted that the frequencies of the longitudinal own vibration of the bar are, like the strings, harmonics in relation to the basic frequency ω_1 (n = 1).

II. TRANSVERSE VIBRATIONS OF THE ROD



The transverse oscillation of the bar with a constant cross-section S and the density ρ along the length l describes the equation:

$$\rho S\frac{\partial^2 z}{\partial t^2}+\frac{\partial^2}{\partial t^2}\left(EI\frac{\partial^2 z}{\partial t^2}\right)=0, (4)$$

where the expression in brackets is the bending moment, while I is the moment of inertia of the cross-section of the rod

$$I=\int_S z^2 dS.$$

Equation (4) is not a wave equation. If we put in a wave solution in formula (4) in the form $z(x,t)=Zexp(j(\omega t-kx))$, we will get a dispersive formula (dispersion - a phenomenon in which the speed of the wave depends on the frequency):

$$\omega=k^2\sqrt{\frac{EI}{\rho S}},$$

where $k=2\pi/\lambda$ is wave number, λ - the length of the transverse wave.
From this formula we can conclude that the speed of moving the surface of a constant phase, witch is the phase velocity of transverse vibrations, is equal to:

$$c=\frac{\omega}{k}=\sqrt{\omega c_L\sqrt{\frac{I}{S}}}\xrightarrow[\omega\to\infty]{}\infty.$$

For physical reasons this is impossible, therefore the equation (4) is not strict. However, for low frequencies for which the transverse wavelength λ is much larger than the linear dimensions of the rod cross-section (a/λ<0.1), equation (4) is sufficiently accurate for technical applications. Substituting for the equation (4): $z(x,t)=Z(x)exp(j\omega t)$, we get:

$$\frac{d^4Z(x)}{dx^4}-\mu^4 Z(x)=0, (5)\ \ \mu^4=\omega^2\frac{\rho S}{EI}.$$

The solution of equation (5) can be presented as:

$$Z(x) = A_1 e^{\mu x} + A_2 e^{-\mu x} + A_3 e^{j\mu x} + A_4 e^{-j\mu x}$$
$$= B_1 cosh\mu x + B_2 sinh\mu x + B_3 cos\mu x + B_4 sin\mu x. (6)$$

The solution of equation (6) contains four constants to determine which four boundary conditions are needed, two at each end of the bar.

The case when one end of the rod is clamped and the other end is free.

For $x = 0$, the deflection and inclination of the bar must be zero:

$$Z(0) = 0 \ and \ \left.\frac{dZ(x)}{dx}\right|_{x=0} = 0.$$

Then $B_1 = -B_3$ and $B_2 = -B_4$.

For $x=l$ the bending moment and the shear force at the free end of the bar must be zero:

$$\left.\frac{d^2Z(x)}{dx^2}\right|_{x=l} = 0 \ \ and \ \left.\frac{d^3Z(x)}{dx^3}\right|_{x=l} = 0.$$

Therefore

$$B_2 = B_1 \frac{sin\mu l - sinh\mu l}{cos\mu l + cosh\mu l} = -B_1 \frac{cos\mu l + cosh\mu l}{sin\mu l + sinh\mu l},$$

where

$$(cos\mu l + cosh\mu l)^2 = sinh^2\mu l - sin^2\mu l,$$
$$cos\mu l \cdot cosh\mu l = -1.$$

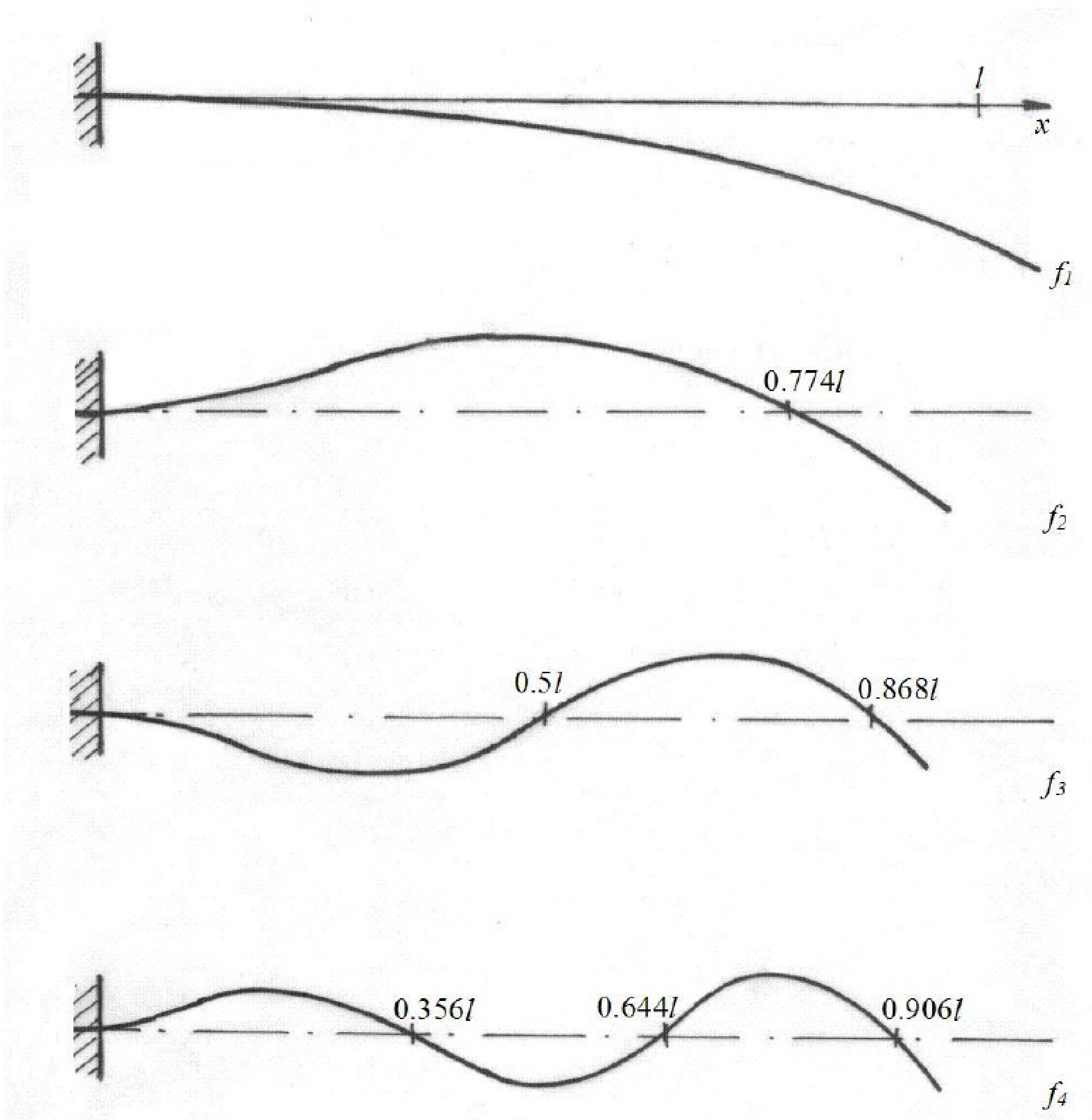
The eigenvalues of the last equation are:

$$\begin{cases} \mu_1 l = 1.8751, \\ \mu_2 l = 4.6946, \\ \mu_3 l = 7.8548, \\ \mu_4 l = 10.9957, \end{cases}$$

For these values μ_n, $n = 1, 2, ...$, we obtain from equation (5) the frequencies of transverse vibrations of the rod:

$$\omega_n^2 = \mu_n^4 \frac{EI}{\rho S}. (7)$$

The next figure shows the first four modes of transverse vibrations of a rod clamped at one end.



Where:

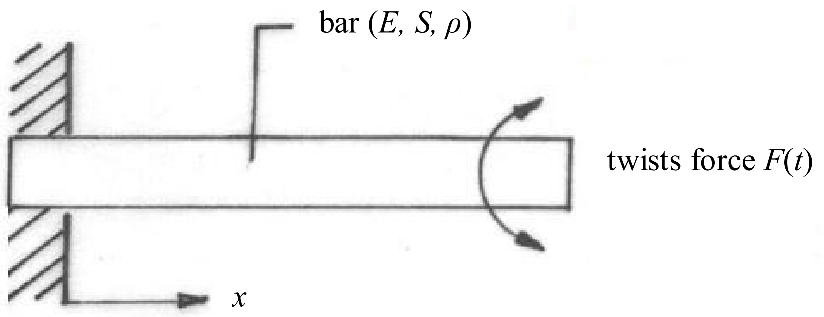
$$f_1 = \frac{0.5596}{l^2}\sqrt{\frac{EI}{\rho S}}, [Hz]$$

$$f_2 = 6.268 f_1,$$

$$f_3 = 17.548 f_1,$$

$$f_4 = 34.387 f_1.$$

III. TORSIONAL VIBRATIONS OF ROD



When the rod is triggered by the torque, torsional (vortex) vibrations arise. The rod that transmits the torsional moments is called the shaft.

The basic frequency of self-oscillating vibrations is given by:

$$f_1 = \frac{1}{2l}\sqrt{\frac{E}{2\rho(1+\sigma)}}, [Hz]$$

where: σ - Poisson's number.

Vibration frequencies of the higher modes are harmonic in relation to the fundamental frequency f_1.